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BURNETT'S MODEL OF THE STRUCTURE OF A SHOCK WAVE IN A MOLECULAR GAS†

V. S. GALKIN and S. V. RUSAKOV

Zhukovskii

e-mail: flumec@progtech.ru

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The Navier–Stokes, Burnett and truncated Burnett systems of equations for a polyatomic gas with a slight exchange of translational and internal energies of the molecules are presented. The establishment method is used to calculate the structure of a shock wave in nitrogen. The results of calculations using these systems of equations are compared with the results of the method of direct statistical modelling. The effect of bulk viscosity and the accuracy of the Eucken approximation in the theory of heat conduction are analysed. The structure of the shock wave in a binary mixture of monatomic gases is calculated using the Navier–Stokes equations and the truncated Burnett equations by numerical upstream integration of a system of ordinary differential equations. The role of thermal diffusion is considered. © 2005 Elsevier Ltd. All rights reserved.

Burnett's equations have been used in many investigations, which are completely or partially reviewed in [1–3], etc. An important stimulus has been the problem of the structure of a strong shock wave, when these equations considerably refine the Navier–Stokes equations. However, the well-known "defects" of the Burnett equations arise, namely, indeterminacy with the boundary conditions (the order of the Burnett system is higher than the order of the Navier–Stokes system), short-wave instability, the presence of "false solutions" and the awkwardness of the equations [1, 2]. To eliminate these, it has been proposed, in particular, to solve the complete system of Burnett equations by the method of successive approximations with respect to a simpler system of the truncated Burnett equations. Investigations have been made of the applicability of the method for a simple gas [2, 4, 5] and for a binary mixture of monatomic gases [3, 6]. It has been shown that the truncated Burnett equations are effective: with a fairly high accuracy, the order of the system is equal to the order of the Navier–Stokes system of equations and there is no short-wave instability.

Below we extend the consideration of this effectiveness. We use results obtained earlier in [7] to formulate Burnett's models for a molecular gas.

1. THE EQUATIONS FOR A POLYATOMIC GAS

The system of Burnett equations has the form

$$\begin{gathered}\frac{\partial\rho}{\partial t}+(\rho u_\alpha)_{,\alpha}=0,\quad \frac{\partial\rho u_\alpha}{\partial t}+(\rho u_\alpha u_\beta+p\delta_{\alpha\beta}+\tau_{\alpha\beta})_{,\beta}=0,\quad p=\rho RT\\ \frac{\partial}{\partial t}\left(\frac{\rho u^2}{2}+\frac{3}{2}pc_v^*\right)+\left[u_\alpha\left(\frac{\rho u^2}{2}+p+\frac{3}{2}pc_v^*\right)+u_\beta\tau_{\alpha\beta}+q_\alpha\right]_{,\alpha}=0\end{gathered}\tag{1.1}$$

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Here

$$\tau_{\alpha\beta} = (\Pi^{(1)} + \Pi^{A} + \Pi^{B})\delta_{\alpha\beta} + \pi^{(1)}_{\alpha\beta} + \pi^{A}_{\alpha\beta} + \pi^{B}_{\alpha\beta}, \quad q_\alpha = q^{(1)}_\alpha + q^{A}_\alpha + q^{B}_\alpha$$
$$c^*_\upsilon = 1 + \frac{2}{3k}c_\upsilon; \quad (\cdot)_{,\alpha} = \frac{\partial(\cdot)}{\partial r_\alpha} \tag{1.2}$$

ρ is the mass density, $\mathbf{u}$ is the gas velocity, $R = k/m$, k is Boltzmann's constant, m is the mass of the molecule, c_υ is the specific heat capacity at constant volume, due to the internal degrees of freedom of the molecules, the components of the radius vector r are introduced by the subscripts α, β and $\gamma = 1, 2, 3$, the rule of summation over repeated subscripts is used, and $\boldsymbol{\delta}$ is the unit tensor. The complete stress tensor

$$\mathbf{P} = p\boldsymbol{\delta} + \boldsymbol{\tau} = (p + \Pi)\boldsymbol{\delta} + \boldsymbol{\pi}$$

where the quantity Π is due to the internal degrees of freedom of the molecules, $\boldsymbol{\pi}$ is the non-divergent stress tensor and $\mathbf{q}$ is the heat flux vector.

Omitting the quantities Π^B, $\pi^B_{\alpha\beta}$, q^B_α in relations (1.1) and (1.2), we obtain the system of truncated Burnett equations retaining only $\Pi^{(1)}$, $\pi^{(1)}_{\alpha\beta}$, $q^{(1)}_\alpha$, the system of Navier–Stokes equations.

In the Navier–Stokes approximation

$$\Pi^{(1)} = -\varsigma\nabla\cdot\mathbf{u}, \quad \pi^{(1)}_{\alpha\beta} = -2\eta e_{\alpha\beta}, \quad e_{\alpha\beta} = \langle u_{\alpha,\beta}\rangle, \quad q^{(1)}_\alpha = -\lambda T_{,\alpha}, \quad \lambda = \lambda_t + \lambda_\upsilon \tag{1.3}$$

Here

$$\nabla\cdot\mathbf{u} = u_{\alpha,\alpha}, \quad \langle A_{\alpha\beta}\rangle = \frac{1}{2}(A_{\alpha\beta} + A_{\beta\alpha}) - \frac{1}{3}A_{\gamma\gamma}\delta_{\alpha\beta} \tag{1.4}$$

ς, η and λ are the bulk and dynamic viscosity coefficients and the thermal conductivity, and the quantities λ_t and λ_υ are due to the translational and internal energies of the molecules respectively.

In the truncated equations, in addition to the terms (1.3), the following Burnett terms are taken into account

$$\Pi^{A} = \omega_1 e_{\alpha\beta}e_{\beta\alpha} + \omega_4(\nabla\cdot\mathbf{u})^2 - \omega_5 u_{\beta,\alpha}u_{\alpha,\beta}$$
$$\pi^{A}_{\alpha\beta} = \xi_1 e_{\alpha\beta}\nabla\cdot\mathbf{u} - \xi_2\langle 2u_{\gamma,\alpha}e_{\gamma\beta} + u_{\gamma,\beta}u_{\alpha,\gamma}\rangle + \xi_3\langle e_{\alpha\gamma}e_{\gamma\beta}\rangle \tag{1.5}$$
$$q^{A}_\alpha = \gamma_1 T_{,\alpha}\nabla\cdot\mathbf{u} + 2\gamma_2\left[\frac{1}{3c^*_\upsilon}T_{,\alpha}\nabla\cdot\mathbf{u} + u_{\beta,\alpha}T_{,\beta}\right] + \gamma_4 T_{,\beta}e_{\beta\alpha}$$

Finally, the terms of the system of Burnett equations

$$\Pi^{B} = \omega_2\nabla^2 T + \omega_3(\nabla T)^2 - \omega_5\left(\frac{p_{,\alpha}}{\rho}\right)_{,\alpha} + \frac{\omega_6}{\rho}p_{,\alpha}T_{,\alpha}$$
$$\pi^{B}_{\alpha\beta} = -\xi_2\left\langle\left(\frac{p_{,\beta}}{\rho}\right)_{,\alpha}\right\rangle + \xi_4\langle(T_{,\alpha})_{,\beta}\rangle + \xi_5\langle T_{,\alpha}T_{,\beta}\rangle + \xi_6\langle p_{,\alpha}T_{,\beta}\rangle \tag{1.6}$$
$$q^{B}_\alpha = 2\gamma_2\frac{T}{3c^*_\upsilon}(\nabla\cdot\mathbf{u})_{,\alpha} + \frac{\gamma_3}{\rho}p_{,\beta}e_{\beta\alpha} + \gamma_5(e_{\alpha\beta})_{,\beta} + \gamma_{10}\frac{p_{,\alpha}}{\rho}\nabla\cdot\mathbf{u} + \gamma_{12}(\nabla\cdot\mathbf{u})_{,\alpha}$$

The Burnett transfer coefficients are given by the following approximate formulae [7]

$$\omega_1 = \frac{2}{p}\eta\varsigma, \quad \omega_2 = \frac{\varsigma}{p}(\lambda_t - \sigma\lambda_\upsilon), \quad \omega_3 = \frac{\varsigma}{p}\left[\frac{d\lambda_t}{dT} - \frac{3}{2}kT^2\frac{d}{dT}\left(\frac{\lambda_\upsilon}{c_\upsilon T^2}\right) - \lambda_\upsilon\frac{2}{3}\frac{\sigma^2}{T}\langle(\Delta\varepsilon_\omega)^3\rangle_c\right]$$
$$\omega_4 = \frac{\varsigma^2}{p}\left\{\frac{5}{2} + \frac{3}{2}\sigma - \frac{2}{3c^*_\upsilon}\left[\frac{3}{2}\partial_T\varsigma + \frac{3}{2}\sigma(\partial_T(\varsigma\sigma) - 2) + \sigma^2\langle(\Delta\varepsilon_\omega)^3\rangle_c\right]\right\} \tag{1.7}$$

$$\omega_5 = \frac{3}{2}\frac{\varsigma^2}{p}(1+\sigma), \quad \omega_6 = 0; \quad \sigma = \frac{3k}{2c_\upsilon}$$

$$\xi_1 = \frac{4}{3}\frac{\eta^2}{p}\left(\frac{7}{2} - \frac{1}{c_\upsilon^*}\partial_T\eta + \frac{3}{2}\frac{\varsigma}{\eta}\right), \quad \xi_2 = \frac{1}{4}\xi_3 = \frac{\eta^2}{p}, \quad \xi_4 = \frac{4}{5}\frac{\eta}{p}\lambda_t, \quad \xi_5 = \frac{4}{5}\frac{\eta}{p}\frac{d\lambda_t}{dT}, \quad \xi_6 = 0 \tag{1.8}$$

$$\gamma_1 = \frac{4m}{15pkc_\upsilon^*}\left\{-\lambda_t^2\partial_T\lambda_t - \frac{5}{3}\lambda_\upsilon\left[T^3\frac{d}{dT}\left(\sigma\frac{\lambda_\upsilon}{T^2}\right) + \frac{2}{3}\lambda_\upsilon\sigma^2\langle(\Delta\varepsilon_\omega)^3\rangle_c\right]\right\} +$$

$$+ \frac{\varsigma}{p}\left\{\lambda_t\partial_T(\varsigma T^2) + \lambda_\upsilon\left[1 - \sigma\partial_T\left(\frac{\varsigma\sigma}{T}\right) - \frac{2}{3}\sigma^2\langle(\Delta\varepsilon_\omega)^3\rangle_c\right]\right\} + \frac{2m}{3kp}\left(\frac{7}{5}\lambda_t^2 + \frac{5}{3}\sigma\lambda_\upsilon^2\right)$$

$$\gamma_2 = -\frac{2m}{5kp}\left(\lambda_t^2 + \frac{5}{3}\sigma\lambda_\upsilon^2\right), \quad \gamma_3 = -\frac{4m\eta}{5kp}\lambda_t \tag{1.9}$$

$$\gamma_4 = \frac{4\eta}{2p}\left\{\lambda_t\partial_T(T^{7/2}\eta) + \frac{5}{2}\lambda_\upsilon + \frac{m}{k\eta}\left(\frac{7}{5}\lambda_t^2 + \frac{5}{3}\sigma\lambda_\upsilon^2\right)\right\}$$

$$\gamma_5 = \frac{4\eta\lambda_t}{5kn}, \quad \gamma_{10} = -\frac{m}{kT}\gamma_{12}, \quad \gamma_{12} = \frac{\varsigma}{nk}(\lambda_t - \sigma\lambda_\upsilon)$$

In formulae (1.5) and (1.6) we have omitted [7] the coefficients ω^*, ξ^* and γ^*, the coefficients ς, η, λ, λ_t and λ_υ defined by expressions (1.3) depend on the temperature, the operation $\langle\ldots\rangle_c$ was postulated earlier in [8], and the operator $\partial_T N$ is given by the formula

$$\partial_T N = d\ln N/d\ln T$$

(for example $\partial_T(\varsigma\sigma) = d\ln(\varsigma\sigma)/d\ln T$).

We will further consider a diatomic gas with excited rotational degrees of freedom of the molecules. The bulk viscosity coefficient is given by the formula [9]

$$\varsigma = \frac{1}{4}\pi k c_\upsilon \eta Z\left(\frac{3}{2}k + c_\upsilon\right)^{-2} \tag{1.10}$$

Here Z is the ratio of the relaxation times of the rotational and translational degrees of freedom. To estimate the value of Z we will use the approximate Parker formula

$$Z = Z_\infty\left[1 + \frac{\pi^{3/2}}{2}\theta^{1/2} + \left(\frac{\pi^2}{4} + 2\right)\theta\right]^{-1}, \quad \theta = \frac{T_*}{T} \tag{1.11}$$

For nitrogen $T_* = 91.5$ K. The results of calculations of the profiles of the gas-dynamic variables in the shock wave for values of $Z_\infty = 15$–22, obtained using the mathematical models in Section 2, differ only slightly (they merge in the scales shown in Fig. 1), and hence we take $Z_\infty = 18.2$.

We will write the thermal conductivity in the form

$$\lambda = \frac{15}{4}R\eta\Lambda, \quad \Lambda = \lambda_t^* + \lambda_\upsilon^* \tag{1.12}$$

where λ_t^* and λ_υ^* are due to the translational and rotational degrees of freedom respectively. In practice one usually confines oneself to the modified Eucken approximation [9, 10], when

$$\lambda_t^* = 1, \quad \lambda_\upsilon^* = \frac{4}{15}\frac{c_\upsilon}{k}\beta_t \approx \frac{4}{15}1.328, \quad \beta_t = \frac{\rho\mathcal{D}}{\eta} \tag{1.13}$$

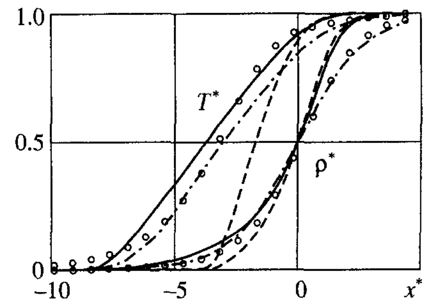

Fig. 1

Here we have taken into account the equality $c_\upsilon = k$; $\mathfrak{D}$ is the coefficient of self-diffusion of the gas, ignoring the rotational degrees of freedom. The quantity β_t depends only slightly on the intermolecular potentials, and hence its average value [10], given in (1.13), is used. At low temperatures the Mason–Monchick approximation [9] is used

$$\lambda_t^* = 1 - \frac{A}{\sigma}, \quad \lambda_\upsilon^* = \frac{2\beta_\upsilon}{5\sigma}(1 + A), \quad \beta_\upsilon = \frac{\rho\mathfrak{D}_\upsilon}{\eta} = \beta_t\varphi(T)$$
$$A = \frac{5 - 2\beta_\upsilon}{\pi Z}\left[1 + \frac{2}{\pi Z}\left(\frac{5}{2\sigma} + \beta_\upsilon\right)\right]^{-1}, \quad \sigma = \frac{3}{2}\frac{k}{c_\upsilon} \tag{1.14}$$

where $\mathfrak{D}$ is the coefficient of self-diffusion of the gas taking into account the rotational degrees of freedom. The ratio $\mathfrak{D}_\upsilon/\mathfrak{D}$ will be estimated using the approximate Sandler formula [9]

$$\mathfrak{D}_\upsilon/\mathfrak{D} \equiv \varphi(T) = 1 + 0.27Z^{-1} - 0.44Z^{-2} - 0.90Z^{-3} \tag{1.15}$$

In what follows we will assume [7]

$$c_\upsilon = k, \quad \sigma = 3/2, \quad \langle(\Delta\varepsilon_\omega)^3\rangle_c = 2 \tag{1.16}$$

The dynamic viscosity coefficient $\eta \sim T^S$, $S = 0.72$.

2. RESULTS OF CALCULATIONS

To solve the problem of the structure of a shock wave using the system of equations described above, we used the numerical establishment method [6]. The method of direct statistical modelling [11] has also been used, generalized to take into account the rotational degrees of freedom using the VRS-model [11, 12] with representation (1.11) for the rotational relaxation time. The results obtained earlier in [12] agree with experimental data on the density profile in a shock wave in nitrogen when $M = 10$ and can serve as standard ones in qualitative considerations. The calculations were carried out for nitrogen at the free stream temperature $T(-\infty) = 100$ K and different Mach numbers M, using formulae (1.10)–(1.12) and (1.14)–(1.16). The reduced temperature, the reduced mass density and the reduce streamwise coordinate are as follows:

$$T^* = \frac{T - T(-\infty)}{T(+\infty) - T(-\infty)}, \quad \rho^* = \frac{\rho - \rho(-\infty)}{\rho(+\infty) - \rho(-\infty)}, \quad x^* = \frac{x}{l}; \quad l = \frac{1}{\sqrt{2}\pi d^2 n}$$

where l is the value of the mean free path upstream of the wave ($x = -\infty$), and the diameter of the "pseudosphere" d of the VHS-model is expressed in terms of its parameters [11] (in formula (2.1) [6] we must put $m_i + m_j$ instead of $m_1 + m_2$).

As previously [2–6], the Burnett models considerably improve the Navier–Stokes model (Fig. 1, $M = 11$, the results obtained using the truncated Burnett equations are represented by the continuous curves, the results obtained using the Navier–Stokes equations are shown by the dashed curves, the results obtained using Burnett's equations are shown by the dash–dot curves and the results obtained using the direct statistical modelling method are represented by the small circles). The agreement

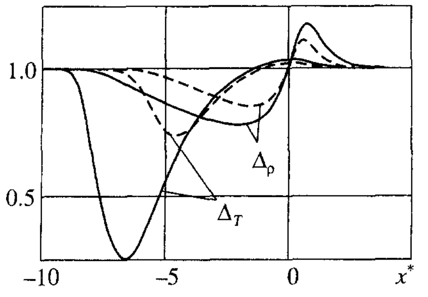

Fig. 2

between the results of a calculation using Burnett's models with the results obtained by direct statistical modelling, are approximately the same as in the case of a monatomic gas, although it was assumed that only slight exchanges occur between the translational and internal energies of the molecules, for which the effect of the bulk viscosity coefficient is substantiated [9, 10]. Previously in [13] a more general relaxation approach was used within the framework of the Burnett equations. In order to improve the accuracy of the theory, a semiempirical equation was developed for the rotational temperature. Expressions which hold in the case of a monatomic gas were employed for the transfer properties.

The approach developed here enables us to estimate the effect of the bulk viscosity coefficient on the profile of the gas-dynamic variables, which is of interest in supersonic aerodynamics: to calculate the flow around bodies the system of Navier–Stokes equations is widely used, but the bulk viscosity coefficient is ignored. When $\varsigma = 0$ the system of equations (1.1)–(1.9) is simplified considerably, since the quantities Π, γ_{10} and γ_{12}, and terms of the expressions for the coefficients ξ_1 and γ_1, proportional to the coefficient ς, vanish.

We will introduce the ratios (for given x^*) of the values of the temperature and density of nitrogen, calculated for $\varsigma = 0$ and $\varsigma \neq 0$,

$$\Delta_T = T(\varsigma = 0)/T(\varsigma \neq 0), \quad \Delta_\rho = \rho(\varsigma = 0)/\rho(\varsigma \neq 0)$$

The results of calculations using the truncated Burnett equations are shown in Fig. 2 for $M = 11$ (the continuous curves), and $M = 5$ (the dashed curves). The deviations of the values of Δ_T and Δ_ρ from unity for the Navier–Stokes equations (but for other x^*) are close to them.

The bulk viscosity has a considerable effect on the profiles of the gas-dynamic variables just upstream of the shock wave. The effect increases as M increases.

We emphasize that, in all the cases considered in this paper, the origin ($x^* = 0$) corresponds to the point where $\rho^* = \frac{1}{2}$.

For gas dynamics the problem of the accuracy of the Eucken approximation (1.12) is fundamental, i.e. the contribution to the solution of the problem of terms of the expressions for the coefficients (1.14), proportional to the quantity $A \sim Z^{-1}$. The value of $\max|T(A = 0)/T(A \neq 0) - 1|$ for $M = 5$ and $M = 11$ is equal to 0.11 and 0.31 in the case of the truncated Burnett equations ($x^* = -5.12$ and $x^* = -7.94$) and 0.03 and 0.12 in the case of the Navier–Stokes equations ($x^* = -2.12$ and $x^* = -3.38$). Hence, the contribution is greater in the case of the truncated Burnett equations, since the quantities λ_t and λ_υ occur in formulae (1.7) and (1.9) separately, and in certain cases quadratically (see (1.9)). In the case of the Navier–Stokes equations these quantities occur in the form of a sum (1.12), where the terms of formula (1.14), proportional to the coefficients A, have opposite signs and partially "cancel" one another. However, the contribution also considered here is not negligibly small in the neighbourhood of the upstream side of the shock wave.

3. A BINARY MIXTURE OF MONATOMIC GASES

The problem of the structure of the shock wave in a simple gas within the framework of the Navier–Stokes equations is solved by numerical upstream integration of the ordinary differential equation [14]. The same procedure can be used in the case of the truncated Burnett equations [2]. However, the nature of the singular points of the complete system of Burnett equations when $M > 1.9$ does not allow it to be used [15], and establishment methods are employed [1, 3, 6, 13].

For a binary mixture, within the framework of the Navier–Stokes equations, the problem can be solved without analysing the phase pattern [16]. We will consider the one-dimensional time-independent truncated Burnett equations, using the relations obtained earlier in [3, 6]. After lengthy calculations we obtain a system of two autonomous ordinary differential equations

$$\frac{df_m}{d\omega} = \frac{K_m(f_1, f_2, \omega)}{L(f_1, f_2, \omega)}, \quad m = 1, 2 \tag{3.1}$$

where f_1 and ω are the dimensionless temperature and dimensionless mean mass velocity, and f_2 is the molar fraction of one of these components. In the case of a simple gas, we have a single equation ($f_2 = 0$), and the analysis is carried out analytically both within the framework of the Navier–Stokes equations [14] and within the framework of the truncated Burnett equations [2]. The curves of $K_1(f_2, \omega) = 0$ and $L(f_1, \omega) = 0$ intersect at the singular points ω_1 and ω_2, corresponding to the value of the dimensionless coordinate $x^* = \pm\infty$. It has been proved that only one integral curve exists, which emerges from the point ω_1 and enters at the point ω_2 (the remaining integral curves leave the region S, bounded by the curves $K_1 = 0$ and $L = 0$ [14]).

In the case of a binary mixture we have a system of two equations (3.1), the quantities K_m and L are given by very lengthy expressions, and hence it is only possible to carry out a computer analysis. In the specific cases considered here, Eq. (3.1) has two singular points: ω_1 and ω_2. The roots of the cubic dispersion equation λ_r ($r = 1, 2, 3$) are real, $\lambda_r > 0$ at the point ω_2 and $\lambda_{1,2} > 0$, $\lambda_3 < 0$ at the point ω_1. Hence, the singular points are three-dimensional analogues of a saddle (ω_1) and of an unstable node (ω_2).

The negative root corresponds to the required integral curve. A computer analysis of the behaviour of the integral curves in the three-dimensional region, similar to the region S in the case of a simple gas, shows that a unique integral curve exists, connecting the points ω_1 and ω_2. Numerical upstream integration of system (3.1) gives the same conclusion. The formulation of the boundary condition at the point ω_1 is analogous to that given earlier in [14, 16].

Our calculations confirmed the results obtained in [3, 6]. It was shown that barodiffusion has only a small effect on the profiles of the gas-dynamic variables (within the framework of the Navier–Stokes equations). We carried out a systematic analysis of the effect of thermal diffusion. In the case of Maxwell molecules we have

$$6C_{12}^* - 5 = 0 \tag{3.2}$$

where C_{12}^* is the ratio of the reduced Ω-integrals [10]. We will call the zero thermal diffusion case approximate when we can formally assume that equality (3.2) is satisfied for any intermolecular potentials. Then, in the notation used previously [7, Section 2], k_T, $\mathcal{D}_T$, Δ and $\delta\gamma_i$ are equal to zero, in which case $\lambda' = \lambda$. The Burnett transfer properties are cardinally simplified: in the notation used previously [3] we will have

$$\Phi_1^h = \sum_{i=1}^{2} \frac{\Omega_i \lambda_i}{4T}\left[\lambda_i \beta_i - G_i T^{-5/2} \frac{\partial(T^{7/2}\eta_i)}{\partial T}\right], \quad \Phi_1^V = \sum_{i=1}^{2} \Psi_i G_i \omega_i \mathcal{D}_{12} \frac{\partial \eta_i}{\partial T}; \quad G_i = \frac{4k}{m_i}$$

instead of the complicated expressions given previously [3, formulae (1.8)].

The effect of thermal diffusion on the profile, i.e. the difference between the data when $k_T \neq 0$ and in the approximation of zero thermal diffusion (3.2), is shown in Fig. 3. The data were obtained using the Navier–Stokes equations for a mixture of molecules in the form of elastic spheres, the diameters of the molecules being the same, and $M = 11$. We have used the following notation [3]

$$X_i^* = \frac{x_i(x^*) - x_i(-\infty)}{x_i(-\infty)}, \quad x_i = \frac{n_i}{n}, \quad c_i = \frac{\rho_i}{\rho}, \quad n = n_1 + n_2, \quad \rho = \rho_1 + \rho_2$$

where n_i, $\rho_i = m_i n_i$ and m_i are the number density, mass density and the mass of a particle of the ith component of the mixture, $i = 1, 2$. The continuous curves were obtained taking thermal diffusion into account, while the dashed curves were obtained ignoring thermal diffusion (i.e. in the approximation (3.2)); the upper two pairs of curves are given for $x_1(-\infty) = 1/2$, and the lower pair is for $c_1(-\infty) = 1/2$. The effect of thermal diffusion in the case when $c_1(-\infty) = 1/2$ is much less than in the case when $x_1(-\infty) = 1/2$.

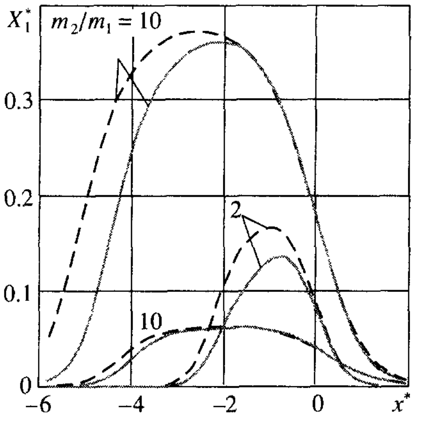

Fig. 3

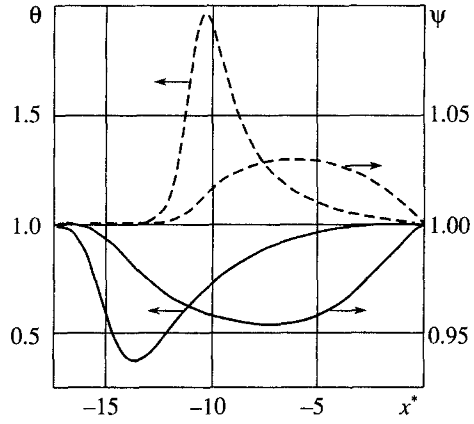

Fig. 4

In Fig. 4 we show the values of the ratios of the temperatures and the mass densities in an argon–helium mixture $(m_2/m_1 \approx 10)$

$$\theta = \frac{T(k_T = 0)}{T(k_T \neq 0)}, \quad \psi = \frac{\rho(k_T = 0)}{\rho(k_T \neq 0)}$$

for the case when $x_1(-\infty) = {}^1\!/_2$, obtained using the truncated Burnett equations (the continuous curves) and the Navier–Stokes equations for $k_T = 0$ (i.e. in approximation (3.2)) and for $k_T \neq 0$.

The effect of thermal diffusion on the temperature profile is much greater than on the density profile. Approximately the same deviations of θ and ψ from unity occur in the case of molecules in the form of elastic spheres with diameter ratios $d_2/d_1 = 1$ and 2. The ratio $m_2/m_1 \geq 1$ is the main factor in the influence of thermal diffusion on the structure of the shock wave. This influence decreases together with m_2/m_1. For example, for an argon-neon mixture $(m_2/m_1 \approx 2)$ the value of $\max|\theta - 1|$ is equal to 0.04 for the truncated Burnett equations and 0.17 for the Navier–Stokes equations. However, it is important to bear in mind that when $m_2/m_1 \geq 10$ it is necessary, strictly speaking, to use the two-fluid description, since the differences in the values of the temperatures and the velocities of the components of the mixture will be more than double [3].

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